

# Macierz Hilberta

$$H_n = \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{n} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{n+1} \\ \vdots & & & \\ \frac{1}{n} & \frac{1}{n+1} & \dots & \frac{1}{2n-1} \end{bmatrix}$$

Zaokrąglona

$$\hat{H}_4 = \begin{bmatrix} 1.0000 & 0.50000 & 0.33333 & 0.25000 \\ 0.50000 & 0.33333 & 0.25000 & 0.20000 \\ 0.33333 & 0.25000 & 0.20000 & 0.16667 \\ 0.25000 & 0.20000 & 0.16667 & 0.14286 \end{bmatrix}$$

Dokładna

$$H_4^{-1} = \begin{bmatrix} 16 & -120 & 240 & -140 \\ -120 & 1200 & -2700 & 1680 \\ 240 & -2700 & 6480 & -4200 \\ -140 & 1680 & -4200 & 2800 \end{bmatrix}$$

zaokrąglona

$$\hat{H}_4^{-1} = \begin{bmatrix} 16.248 & -122.72 & 246.49 & -144.20 \\ -122.72 & 1229.9 & -2771.3 & 1726.1 \\ 246.49 & -2771.3 & 6650.1 & -4310.0 \\ -144.20 & 1726.1 & -4310.0 & 2871.1 \end{bmatrix}$$

$$\begin{aligned} \text{cond}(H_4) &= \|H_4\| \|H_4^{-1}\| = \frac{25}{12} \cdot 13620 \\ &= 28000 \end{aligned}$$