UKŁADY TRÓJDIAGONALNE

$$A = \begin{bmatrix} b_1 & c_1 & 0 & 0 & \cdots & & 0 \\ a_2 & b_2 & c_2 & 0 & & & 0 \\ 0 & a_3 & b_3 & c_3 & & & 0 \\ \vdots & & \ddots & & & & \vdots \\ & & & & a_{n-1} & b_{n-1} & c_{n-1} \\ 0 & & \cdots & & 0 & a_n & b_n \end{bmatrix}$$

wtedy

$$LU = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ \alpha_2 & 1 & 0 & & 0 \\ 0 & \alpha_3 & 1 & & \\ \vdots & & & \ddots & \vdots \\ 0 & & \cdots & \alpha_n & 1 \end{bmatrix} \cdot \begin{bmatrix} \beta_1 & c_1 & 0 & & \cdots & 0 \\ 0 & \beta_2 & c_2 & 0 & & 0 \\ \vdots & & \ddots & & & \vdots \\ & & & & \beta_{n-1} & c_{n-1} \\ 0 & & \cdots & & 0 & \beta_n \end{bmatrix}$$

Równania

$$\begin{aligned} \beta_1 &= b_1 & &: \quad 1 \text{ of } LU \\ \alpha_2\beta_1 = a_2,\ \ \alpha_2 c_1 + \beta_2 &= b_2 & &: \quad 2 \text{ of } LU \\ \alpha_j\beta_{j-1} = a_j,\ \ \alpha_j c_{j-1} + \beta_j &= b_j & &: \quad \text{w } j \text{ of } LU \end{aligned}$$

wiersz

$j = 3, \ldots, n$.

Proste do rozwiązania

$$\beta_1 = b_1$$
$$\alpha_j = a_j/\beta_{j-1}, \qquad \beta_j = b_j - \alpha_j c_{j-1}, \qquad j = 2, \ldots, n$$

Teraz rozwiązujemy $Lg = f$ (do przodu)

$$g_1 = f_1$$
$$g_j = f_j - \alpha_j g_{j-1}, \qquad j = 2, 3, \ldots, n$$