Układy równań liniowych

$$a_{11} x_1 + a_{12} x_2 = b_1$$

$$a_{21} x_1 + a_{22} x_2 = b_2$$

Ogólnie

$$a_{1,1}x_1 + a_{1,2}x_2 + a_{1,3}x_3 + \cdots + a_{1,n}x_n = b_1$$
$$a_{2,1}x_1 + a_{2,2}x_2 + a_{2,3}x_3 + \cdots + a_{2,n}x_n = b_2$$
$$\vdots$$
$$a_{n,1}x_1 + a_{n,2}x_2 + a_{n,3}x_3 + \cdots + a_{n,n}x_n = b_n$$

lub

$$A x = b$$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \ldots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \ldots & a_{2n} \\ & & \vdots & & \\ a_{n1} & a_{n2} & a_{n3} & \ldots & a_{nn} \end{bmatrix}, \quad b = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}, \quad x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

PRZYKŁAD

$$x_1 + 2x_2 + 3x_3 = 1$$
$$2x_1 + 2x_2 + 3x_3 = 1$$
$$3x_1 + 3x_2 + 3x_3 = 1$$

$$\operatorname{rank}[A \vdots b] = \operatorname{rank} A$$

$$\det A = 3 \neq 0$$

$$\operatorname{rank}[A \vdots b] = 3$$

$$x_1 = x_2 = 0, \qquad x_3 = \tfrac{1}{3} \quad \blacksquare$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 2 & 3 \\ 3 & 3 & 3 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad x = \begin{bmatrix} 0 \\ 0 \\ 1/3 \end{bmatrix} \quad \blacksquare$$